

An inhomogeneous fractal cosmological model

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Abstract.

We present a cosmological model in which the metric allows for an inhomogeneous Universe with no intrinsic symmetries (Stephani models), providing the ideal features to describe a fractal distribution of matter. Constraints on the metric functions are derived using the expansion and redshift relations and allowing for scaling number counts, as expected in a fractal set. The main characteristics of such a cosmological model are discussed.

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1. Introduction

One of the most interesting results of observational cosmology over the last two decades has been the discovery of large scale structures and voids in matter distribution.

To date, however, a clear definition of an homogeneity scale in matter distribution is still lacking. Moreover, this controversy concerns the morphological features of the galaxy clustering, which has been found by several authors to have fractal characteristics at least up to a certain scale (Sylos Labini, Montuori & Pietronero 1998; Wu et al. 1999). In this case, a suitable cosmological model is lacking, given that the Cosmological Principle cannot be applied.

Many authors have attempted to develop alternative models, following two different approaches. The first relies on a perturbative scheme, through the superposition of a fractal density perturbation to the FRW background, both in Newtonian gravity (Joyce et al. 2000) and in general relativity (e.g. Mittal & Lohiya 2001). The second concerns a more general relativistic study of the metric, used to match a fractal number counts relation, and extensive analytical (e.g. Humphreys et al. 1998) and numerical (Ribeiro 1992) computations have been performed.

In any case, for both approaches, only Lemaitre-Tolman-Bondi (LTB) inhomogeneous metric has been considered, i.e. spherically symmetric solutions to Einstein field equations.

Even if this is certainly a very interesting study, it is not the most suitable tool to study a fractal distribution.

As a matter of fact, in a fractal set of points the number density decreases on average from any point of the set. In a spherically symmetric distribution around a point, this is true only if you measure the density centering on that point.

In this paper, the above restriction is relaxed and an inhomogeneous metric with no intrinsic symmetries is presented.

The starting metric is recovered from the class of Stephani solutions (Stephani 1967). Specifically, the aim of this work is to extend the analytical characterization of a fractal matter distribution to a Stephani Universe, whose geometrical (e.g Krasinski 1983) and thermodynamical (e.g. Quevedo & Sussmann 1995) description has already been widely discussed and will not be pursued here.

In Section 2 the general properties of the Stephani model are recalled while the consequences on the redshift expression and expansion are presented in Section 3. In Section 4 we derive the fractal constraints on the metric functions and the main results are discussed in Section 5.

Throughout the paper we have assumed natural units ($G = c = 1$) and the metric signature $(+,-,-,-)$. The Greek indices refer to the whole components, while the Latin letters are just for the spatial part.

2. The model

The metric of the Stephani Universe can be written as follows (Krasinski 1997):

$$ds^2 = D^2 dt^2 - \frac{R^2(t)}{V^2} (dx^2 + dy^2 + dz^2) \quad (1)$$

where :

$$V = 1 + \frac{1}{4}k(t)\{[x - x_0(t)]^2 + [y - y_0(t)]^2 + [z - z_0(t)]^2\} \quad (2)$$

$$D = F(t) \left(\frac{\dot{V}}{V} - \frac{\dot{R}}{R} \right) = F \frac{R}{V} \frac{\partial V}{\partial t} \quad (3)$$

$$k(t) = [C^2(t) - 1/F^2(t)]R^2(t) \quad (4)$$

C, F, R, x_0, y_0, z_0 are arbitrary functions of time and dots denote time derivative .

Eq.(4) is equivalent to the Friedmann equation, although it must be stressed that the analogy is only formal and dictated by the same derivation (through Einstein field equations), but the function $R(t)$ is not the equivalent of the scale factor $a(t)$ in FRW models.

As can be seen from eq.(1) and more precisely from the expression for V in eq.(2), the model is inhomogeneous and has no intrinsic symmetries, unless x_0, y_0, z_0 are strictly constant. In this case x_0, y_0, z_0 become the center of spherical symmetry, even if the metric is still not homogeneous.

Moreover, the distinction between open and closed Universe is more subtle than in FRW;

indeed $k(t)$ is a function of time and so it is not fixed as in FRW model. However, it is worth remembering that fixing $k(t)$ is not a general requirement of Einstein's theory of gravitation.

Let us consider a perfect fluid; the stress-energy tensor is:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}. \quad (5)$$

The Stephani metric (Eq.(1)) fulfills Einstein field equations if the matter fluid has no shear ($\sigma = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = 0$), no rotation ($\omega = \frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu} = 0$) and mass is flowing along t-coordinate lines with:

$$u^\mu = \frac{V}{F} \frac{R}{R\dot{V} - V\dot{R}} \delta_0^\mu \quad (6)$$

$$\dot{u}^i = \frac{V^2}{DR^2} D_{,i}, \quad \dot{u}^0 = 0 \quad (7)$$

$$\theta \equiv u^\mu_{;\mu} = -\frac{3}{F} \quad (8)$$

where δ^μ_ν is the Kronecker symbol. u^μ is the 4-velocity, the notation $_{,i}$ means the spatial derivative and $_{;\mu}$ the covariant derivative. In the FRW model, $\theta = 3H$ where H is the *Hubble constant*.

It is important to note that in the model the matter does not flow along geodesics, as follows from eq.(7); this is a significant difference with FRW model (which have $\dot{u}^i = 0$) (Krasinski 1983).

Turning to thermodynamics properties, Einstein field equations with a perfect fluid source, the conservation law ($T^\mu_{;\nu} = 0$) and the contracted Bianchi identities coupled to the matter conservation equation ($nu^\mu_{;\mu} = 0$) can be reconciled in an equation of state, if the thermodynamical quantities are defined as follows:

$$8\pi\rho(t) = 3C^2(t) \quad (9)$$

$$8\pi p(t) = -3C^2(t) + 2C^2(t)C_{,t}\frac{V}{R}\left(\frac{\dot{V}}{V} - \frac{\dot{R}}{R}\right)^{-1} \quad (10)$$

$$T = \frac{3(p + \rho)}{n^{4/3}} \quad (11)$$

$$S = S_0 + \sigma, \sigma \equiv -\frac{F}{V} \quad (12)$$

satisfying the Gibbs-Duhem equation (Quevedo & Sussman 1995):

$$dS = \frac{1}{T} \left[d\left(\frac{\rho}{n}\right) + p d\left(\frac{1}{n}\right) \right]. \quad (13)$$

As underlined by other authors (Krasinski, Quevedo & Sussman 1997), the physical meaning of the equation of state restricts the freedom in choosing the metric functions, though a defined correspondence among their properties and the physical quantities cannot be set, in general. However, since this work is mainly focused on a fractal model characterization, we will assume that the thermodynamic scheme holds and the metric

constraints will be derived independently.

Again, a deep difference with respect to FRW models emerges, namely the equation of state is not barotropic, as it is evident from Eq.(9) and Eq.(10).

The fact that the equation of state depends on position, as indicated by eq.(9)-(12), and the absence of any intrinsic symmetry make the model particularly suited in the description of a fractal distribution of matter.

3. The redshift equation and the expansion flow

By definition, the redshift Z is determined along a photon path (written in terms of the affine parameter λ) as follows (Ellis et al. 1985):

$$(1 + Z) \equiv \frac{(u^\mu k_\mu)_{em}}{(u^\mu k_\mu)_{obs}} = \left(\frac{dt}{d\lambda} \right)_{em} / \left(\frac{dt}{d\lambda} \right)_{obs}. \quad (14)$$

On the other hand, the definition of redshift refers to the ratio of the time shift along observer's world line with respect to the shift along a general world line, namely:

$$(1 + Z) \equiv \frac{dt}{d\tau}. \quad (15)$$

A more formal manner to find the aforementioned relation is given by considering the photon wave vector k^μ , which satisfies:

$$k^\mu \equiv t_{,\mu} = \frac{dx^\mu}{d\lambda} \quad (16)$$

$$k^\mu k_\mu = 0 \quad (17)$$

therefore, with no loss of generality, normalizing with respect to the observer's value, it yields:

$$(1 + Z) = u_{em}^0 = \frac{1}{D} = \frac{V}{F} \frac{R}{R\dot{V} - V\dot{R}}. \quad (18)$$

It is worth noting that the above *redshift equation* is dependent from the position, which is another important difference with respect to FRW models.

In addition, other authors (Ribeiro 1992) have noted that more information about photon motion can be gained by evaluating the Lagrangian of the metric in eq.(1) and then solving for the Euler-Lagrange equations:

$$\frac{d}{d\lambda} \frac{\partial L}{\partial \dot{x}^\mu} - \frac{\partial L}{\partial x^\mu} = 0, \quad \dot{x}^\mu = \frac{dx^\mu}{d\lambda}. \quad (19)$$

Inserting the past light cone expression relating the derivatives of the time and spatial parts of the metric (as follows by setting $ds^2 = 0$) into the Euler-Lagrange equation for the time component, a few algebra leads to the time motion of photons:

$$\frac{d^2 t}{d\lambda^2} + \left[\frac{\dot{D}}{D} + \frac{\dot{D}}{D^3} - \frac{\dot{R}}{RD^2} + \frac{\dot{V}}{VD^2} \right] \left(\frac{dt}{d\lambda} \right)^2 = 0. \quad (20)$$

In the motion of a photon the affine parameter λ is equivalent to the proper time τ , so that, from eq.(15) and eq.(18), eq.(20) gives the *redshift constraint* on the metric:

$$\frac{\dot{D}}{D} + \frac{\dot{V}}{V} - \frac{\dot{R}}{R} = 0 \quad (21)$$

or equivalently:

$$\frac{\dot{D}}{D^2} + \frac{1}{F} = 0. \quad (22)$$

Comparing the V, R, F dependence in the redshift equation (eq.18) with the expansion expression in eq.(8) and including the redshift constraint (eq.(21)), yields:

$$\frac{\theta}{3} = -\frac{1}{(1+Z)} \frac{\partial}{\partial \tau} (1+Z). \quad (23)$$

The left-hand side of the above equation is the equivalent of the Hubble parameter in the FRW models ($3H = \theta$). In this case, the corresponding relation is:

$$H_{FRW} \equiv \frac{\dot{a}}{a} = -\frac{1}{(1+Z)} \frac{d}{dt} (1+Z) \quad (24)$$

where $a(t)$ is the scale factor.

Though a close correspondence between the two expression is apparent, they are quite different. In the case of an inhomogeneous model, the expansion is linked to the matter distribution since $(1+Z)$ (eq.(18)) depends on the position via the function V .

In other words, different regions of the Universe can undergo different expansion histories.

This is evident from the expansion law, which appears to contain an additional term. Indeed, the general redshift-distance relation is (Ehlers 1993):

$$Z = \left(\frac{\theta}{3} + \sigma_{\mu\nu} e^\mu e^\nu \right) \delta l - \dot{u}_\mu \delta_\perp x^\mu \quad (25)$$

where e^μ is the unit vector joining the observer and the emitting source, whose distance is δl , and directed towards the source. The position vector $\delta_\perp x^\mu$ is defined in the following way. Let h^μ_ν be the tensor projecting the tangent vector-space at each point perpendicularly onto the three dimensional subspace orthogonal to u^i , namely:

$$h^\mu_\nu = \delta^\mu_\nu + u^\mu u_\nu. \quad (26)$$

Considering a displacement δx^ν , we can define the position vector as:

$$\delta_\perp x^\mu = h^\mu_\nu \delta x^\nu \quad (27)$$

and then:

$$\delta l = (g_{\mu\nu} \delta_\perp x^\mu \delta_\perp x^\nu)^{1/2}. \quad (28)$$

The shear contribution vanishes in the present model, therefore, by using the *redshift constraint* and introducing the acceleration in terms of the metric functions (eq.(7)), eq.(25) can be written as:

$$Z = \frac{\dot{D}}{D^2} \delta l + \frac{D_{,i}}{D} \delta x^i. \quad (29)$$

where $\frac{D_{,i}}{D}\delta x^i$ is the kinematic acceleration evaluated at the position fixed by δl .

The first term is the analogous to FRW Hubble term (albeit it depends on the position), since $\frac{\dot{D}}{D^2} = H$ and δl is the comoving line element (see next section, eq.(40)). On the other hand, the additional second term is a new dipolar one, that is purely cosmological, i.e. not due to peculiar velocities.

A few consequences of this will be further analyzed in the last section, nevertheless, it is worth noting that the present model is suitable to describe a fractal distribution, since all the cosmological parameters depend on the position.

Using eq.(18) and eq.(29), the metric function D must satisfy the following differential equation:

$$\frac{\dot{D}}{D^2}\delta l + \frac{D_{,i}}{D}\delta x^i - \frac{1}{D} + 1 = 0 \quad (30)$$

and its solution can provide the space-time expression for the redshift.

Eq.(30) basically describes a nonlinear wave motion with a damping factor. For instance, if the one dimensional equivalent is considered, eq.(30) has the general shape of the so-called *Burger's equation*, which is an approximate description of one dimensional turbulence, with a damping term.

4. Fractal constraints

The density of sources along the δl direction after an affine parameter displacement ($d\lambda$) at some point P is (Ribeiro 1992):

$$d^2 N = \delta l_0^2 d\Omega_0 [n(-k^\mu u_\mu)]_P d\lambda \quad (31)$$

where δl_0 is the observer area distance ($\delta l_0^2 = \frac{dS_0}{d\Omega_0}$) and n is the density of radiating sources in a subtended angle $d\Omega_0$.

The density of sources can be found through the proper luminous matter density, assuming that the basic structures are galaxies with nearly the same mass M_G :

$$n = \frac{\rho_m}{M_G} = \frac{3C^2(t)}{8\pi M_G}. \quad (32)$$

Using eq.(31) and eq.(32), a comoving observer measures a number of sources corresponding to:

$$N(\delta l_0) = \frac{3\delta l_0^2}{2M_G} \int_{\Delta t} C^2(t) dt. \quad (33)$$

where integration is performed over the time Δt spent by the photon along its path; on the other hand, as a more physical interpretation, it directly links the motion of the photon to the intervening matter distribution, as the expression in eq.(9) for $C^2(t)$ shows.

This value can be matched to the expected fractal value in the following way.

Averaging over a sphere centered at the observer location, a volume and a volume density can be defined:

$$V(d_l) = \frac{4}{3}\pi d_l^3 \quad (34)$$

$$\rho_v(d_l) = \frac{M_G N(\delta l_0)}{V(d_l)} \quad (35)$$

where this time the luminosity distance d_l corresponding to δl_0 was introduced, since it is the observational distance measure. It can be related to the observer area distance by means of the redshift factor $(1 + Z)$ through the following:

$$d_l = \left(\frac{dS_0}{d\Omega_0} \right)^{1/2} (1 + Z)^2 = \delta l_0 (1 + Z)^2. \quad (36)$$

A fractal distribution of matter is characterized by a power-law scaling[†]:

$$N(d_l) = N_0 \left(\frac{d_l}{l_h} \right)^{3-\gamma} \quad (37)$$

$$\rho_v(d_l) = \frac{3N_0 M_G}{4\pi l_h^3} \left(\frac{d_l}{l_h} \right)^{-\gamma} \quad (38)$$

where l_h is a transition scale corresponding to homogeneity and $d_f = 3 - \gamma$ is the fractal dimension.

From the above equations and using the measured number counts, it yields:

$$I_\gamma(\delta l_0, Z) \equiv \int_{\Delta t} C^2(t) dt = \frac{2}{3} \frac{N_0 M_G}{l_h^2} \left(\frac{\delta l_0}{l_h} \right)^{1-\gamma} (1 + Z)^{2(3-\gamma)} \quad (39)$$

which links the unknown metric functions with observable quantities N_0, M_G, γ, l_h and is the *fractal constraint* on the metric. According to eq.(9), the metric function $C(t)$ is related to density evolution $\rho(t)$, so that the physical meaning of the fractal constraint is to relate redshift and matter distribution in the proper way over each $t = \text{const}$ hypersurface at each position.

Evaluation of the metric terms at the homogeneity scale ($I_\gamma(l_h, Z(l_h))$) simplifies the above relations and involves the knowledge of the fractal parameters (γ, l_h), the number of galaxies N_0 and the typical galactic mass (M_G), which are provided by the analysis of the galaxy catalogues, and the redshift factor, whose expression can be inserted from the solution of eq.(30).

Otherwise, the value of δl_0 must be obtained using the proper-distance aperture of the subtended area, which follows from the line element for the metric in eq.(1). The line element along the i -th axis at a fixed time t_0 is therefore:

$$\delta x_0^i(t_0) = \int_0^{\delta x_0^i} \frac{R(t_0)}{V(t_0, x_0^i)} dx^i \quad (40)$$

and can be found as (Krasinski 1983):

$$2R(t_0)[k(t_0)]^{-1/2} \arctan \left(\frac{1}{2}[k(t_0)]^{1/2} \delta x_0^i \right) \quad k(t_0) > 0 \quad (41)$$

$$R(t_0) \delta x_0^i \quad k(t_0) = 0 \quad (42)$$

$$\frac{R(t_0)}{[|k(t_0)|]^{1/2}} \ln \frac{1 + \frac{1}{2}[|k(t_0)|]^{1/2} \delta x_0^i}{1 - \frac{1}{2}[|k(t_0)|]^{1/2} \delta x_0^i} \quad k(t_0) < 0. \quad (43)$$

[†] This scaling law should strictly apply only in an Euclidean space-time, since no general relativistic extension to it has been derived, so far. However, it has gained observational support, so it is used here without any theoretical speculations.

In addition, the value of N_0 can be obtained by setting the matching condition to a FRW metric as l_h is approached (Humphreys et al. 1998), i.e. by equating the expression for $N(\delta l)$ evaluated at l_h with the FRW analogous expression at the corresponding radial comoving homogeneity scale r_h (Coles & Lucchin 1995):

$$N_{FRW}(r_h, k_{FRW}) = 4\pi \int_0^{r_h} \frac{n[t(r)]a[t(r)]r^2}{(1 - k_{FRW}r^2)^3} dr \quad (44)$$

in terms of the FRW scale factor $a(t)$ and curvature constant $k_{FRW} = -1, 0, +1$. The above relation can be expanded and restated using the number counts continuity ($na^3 = n_0a_0^3$), if the number of particles in the lapse time $t_0 - t(r)$ is kept fixed (i.e. no particle creation/destruction and evolution), as follows (Coles & Lucchin 1995):

$$N_{FRW}(r_h, k_{FRW}) \simeq 4\pi n_0 a_0^3 \left(\frac{r_h^3}{3} - \frac{1}{10} k_{FRW} \delta l^5 + O \right). \quad (45)$$

The condition:

$$N_{FRW}(r_h, k_{FRW}) = N(l_h, k(t)) \quad (46)$$

provides:

$$N_0 = 4\pi n_0 a_0^3 \left(\frac{r_h^3}{3} - \frac{1}{10} k_{FRW} r_h^5 + O \right). \quad (47)$$

It should be emphasized that eq.(46) and eq.(47) allow a comparison between the basic FRW parameters (a_0, k_{FRW}) and the ones involved in the present model, as they should represent the large scale structure in an equivalent way at the homogeneity scale.

Although the *fractal constraints* provides a link among metric functions, it is more appealing to infer an analogous expression depending on the luminosity distance, being it the proper observational distance measure.

In this case, the same route can be followed, but an important modification at the very beginning must be set. As discussed by other authors (Celerier & Thieberger 2001), if the observer area $S(d_a)$ expressed in terms of the angular distance d_a and the area $S(d_l)$ resulting from the luminosity distance d_l are matched, then, using eq.(36), it yields:

$$S(d_l) = S(d_a)(1 + Z)^4. \quad (48)$$

Again, considering eq.(36) and the solid angles $\Omega(d_a)$, $\Omega(d_l)$ respectively corresponding to $S(d_a)$, $S(d_l)$, by differentiating the above equation, an *aberration* effect on the solid angle is found:

$$\Omega(d_l) = \Omega(d_a) \left[1 - 4 \int_{\Delta t} \frac{\theta/3}{(1 + Z)} dt \right] \equiv \Omega(d_a) A \left(\frac{\theta}{3}, Z \right) \quad (49)$$

and the *aberration function* $A \left(\frac{\theta}{3}, Z \right)$ introduced, as can be seen, describes the effect of the expansion on the subtended angle, i.e. the integration of expansion in proper time provides the aberration of the solid angle.

Using such a condition, the number counts in eq.(33) can be expressed in terms of the luminosity distance. In general, the latter is not a well defined quantity when referring

to cross sectional area perpendicular to light rays (δl_0^2 in eq.(31). The actual value of $N(d_l)$ can be obtained by inserting eq.(36) and eq.(49) in eq.(31), then:

$$N(d_l) = \frac{3d_l^2}{2M_G A\left(\frac{\theta}{3}, Z\right)} \int_{\Delta t} \frac{C^2(t)}{(1+Z)^4} dt. \quad (50)$$

By defining:

$$I_\gamma(d_l, Z) \equiv \int_{\Delta t} \frac{C^2(t)}{(1+Z)^4} dt \quad (51)$$

we get the new *fractal constraint*:

$$I_\gamma(d_l, Z) = \frac{2}{3} \frac{N_0 M_G}{l_h^2} A\left(\frac{\theta}{3}, Z\right) \left(\frac{d_l}{l_h}\right)^{1-\gamma} \quad (52)$$

which indeed is now dependent on luminosity distance, redshift and expansion, namely the observational quantities.

5. Discussion and conclusions

The motivation for this work has been to describe a cosmological inhomogeneous model, which could represent a fractal distribution. Since spherical symmetry does not completely describe a fractal distribution, we have considered Stephani models, i.e. inhomogeneous models with no intrinsic symmetries.

Our results are the following.

First, the redshift expression depends on the position; of course, this character is present in any related cosmological parameter, such as the expansion and the Ω value.

In the FRW, expansion is given by the Hubble parameter, which is the only term governing the Hubble law. On the converse, in the present model, the Hubble law has an additional dipole term due to acceleration, which vanishes in the standard homogeneous cosmology (the quadrupole term due to shear vanishes as in FRW). Usually, the observed dipole term is ascribed to peculiar velocities; in the present model would add a pure cosmological contribution to it.

The existence of a kinematic acceleration in inhomogeneous models has been invoked by other authors (Pascual-Sanchez 2000) as the origin of accelerated expansion of the Universe, as found by recent SNaI Ia data (Perlmutter et al. 1999; Riess et al. 1999). This could be considered as a viable alternative to the proposed interpretation within FRW scheme, namely a positive cosmological constant or vacuum energy or quintessence. Therefore, the striking implication of the coupling of space and time dependences in redshift is that different regions of the Universe could undergo different expansion histories.

Actually, it should be stressed that the present model would lead to a deeply new interpretation of observations and many cosmological and astrophysical contexts should be reconsidered.

A second result is that we have explicitly introduced a fractal characterization of the model.

The density depends on the redshift, which depends on the position. In addition, it fulfills the scaling law imposed by the fractal distribution. Then, the density changes according to the fractal spatial scaling law and the redshift. This approach allows to map the density evolution on $t = \text{const}$ hypersurfaces, applying the fractal constraint. The homogeneity scale l_h plays an important role in the model. We recall that it is the scale at which the Stephani model should match the FRW one, i.e. the scale at which homogeneity and isotropy of the Universe are recovered. The greatest information supply should come by estimating the parameters at that threshold length and imposing the FRW/fractal matching condition.

Summarizing the whole set of constraints, the present model can explicitly express D , F and C from functions in the metric.

The key information still missing concerns probably the most extreme assumption of the model $k(t)$, i.e. curvature time evolution, that is no more kept fixed and bound to -1, 0, +1 values as in FRW Universe. Once $k(t)$ is specified, R and V can be fully determined and a self-consistent cosmology can be built.

So far, we have listed the main differences between the FRW model and the one proposed here. However, an important point against inhomogeneous models is their incompatibility with high degree of isotropy of the Cosmic Microwave Background (CMB). Indeed, it was demonstrated that an isotropic thermal radiation in an expanding Universe implies a FRW cosmology (*EGS theorem*, Ehlers, Green & Sachs 1968). The point is that the *EGS theorem* was demonstrated for geodetic and non-accelerating observers, which is not the case in Stephani models. If acceleration is not vanishing, an inhomogeneous cosmology allows a thermal radiation with high degree of isotropy as observed (Ferrando, Morales & Portilla 1992) and specific Stephani solutions can be built without any violation to the CMB restriction (Clarkson & Barret 1999).

Nevertheless, it is to stress that limitations to the degree of inhomogeneity of the model must be compatible with, for instance, CMB isotropy and Hubble law local trend; for such a purpose, the derived *redshift* and *fractal constraints* must be tuned.

Actually, a more subtle and rather philosophical argument comes into play when facing with any inhomogeneous cosmology, that is the reliability and interpretation of the Cosmological Principle.

Moreover, the present model poses serious questions about the Copernican Principle, as well, which states that all observers are equivalent in the Universe. This is the basis, for instance, of the *EGS theorem* and is far from being straightforwardly fit by a fractal structure.

A last comment concerns the observations, most of which can be reproduced by FRW cosmology. As a conservative statement, it could be noted that each Stephani solution includes FRW as a limiting case, so that observations cannot in general rule them out. The motivation for keeping FRW as a description of the Universe is actually that they are the easiest theoretical model. This led to prefer any modification to come from outside the cosmological framework (e.g. inflation, dark matter) and not in the cosmology itself. Therefore, the theoretical choice has been overwhelming any observational basis, whose

actual interpretation is not *a priori* excluded and could be reanalyzed within the class of inhomogeneous models.

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References

- Celerier M.-N. and Thieberger R 2001 *Astron. Astrophys.* **367** 449
- Clarkson C A and Barret R K 1999 *Class. Quantum Grav.* **16** 3781
- Coles P and Lucchin F 1995 *Cosmology* (New York: John Wiley & Sons)
- Ehlers J 1993 *Gen. Rel. Grav.* **25** 1225, English translation from *Akad. Wiss. Lit. Moinz. Abhandl Math. Nat. Kl.* **11** 793 (1961) **9** 1344
- Ehlers J, Green P and Sachs R K 1968 *J. Math. Phys.* **9** 1344
- Ellis G F R, Nel S D, Maartens R, Stoeger W R and Whitman A P 1985 *Phys. Rep.* **124** 315
- Ferrando JJ, Morales J A and Portilla M 1992 *Phys. Rev. D* **46** 578
- Humphreys N P, Matravers D R and Maartens R 1998 *Class. Quantum Grav.* **15** 3781
- Joyce M, Anderson P W, Montuori M, Pietronero L, Sylos Labini F 2000 *Europhys. Lett.* **49** 416
- Krasinski A 1983 *Gen. Rel. and Grav.* **15** 673
- Krasinski A, Quevedo H and Sussman R A 1995 *J. Math. and Phys.* **38** 20602
- Krasinski A 1997 *Inhomogeneous cosmological models* (Cambridge: Cambridge University Press)
- Mittal A K and Lohiya D 2001 preprint astro-ph/0104370
- Pascual-Sanchez J-F 2000 *Class. Quantum Grav.* **17** 4913
- Perlmutter S et al. 1999 *Astrophys. J.* **517** 565
- Quevedo H and Sussman R A 1995 *J. Math. Phys.* **36** 1365
- Ribeiro M B 1992 *Astrophys. J.* **388** 1
- Riess A G et al. 1999 *Astron. J.* **119** 1009
- Sylos Labini F, Montuori M and Pietronero L 1998 *Phys. Rep.* **293** 66
- Stephani H 1967 *Comm. Math. Phys.* **4** 137
- Wu K.K.S, Lahav O. and Rees M. 1999 *Nature* **397** 225